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MA–101/1841

B. Tech. (EI) (First Semester)

EXAMINATION, 2019

MATHEMATICS—I

Time : Three Hours

Maximum Marks : 100

Note : Attempt questions from both Sections as directed.

Section—A

(Short Answer Type Questions)

Note : Attempt any *ten* questions. Each questions carries 4 marks. 10×4=40

1. If $z = \dfrac{(x^2 + y^2)}{x+y}$, show that :

$$\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$$

2. Use Maclaurin's theorem to prove :

$$e^x \sec x = 1 + x + \frac{2x^2}{2!} + \frac{4x^3}{3!} + \ldots..$$

3. Find the points of inflexion of the curve :

$$y(a^2 + x^2) = x^3$$

4. Show that every square matrix can be uniquely expressed as a sum of a Hermitian and a skew Hermitian matrice.

5. Find the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 5 \\ 1 & 3 & 4 \end{bmatrix}$$

6. Find whether the set of vectors :

$$\{(3, 2, 4), (1, 0, 2), (1, -1, -1)\}$$

are linearly independent.

7. If $x = r\cos\theta$, $y = r\sin\theta$, show that :

$$\frac{\partial(r,\theta)}{\partial(x,y)} = \frac{1}{r}$$

8. If :

$$\mu = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right),$$

prove that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\cot u$$

9. Find the stationary points of the functions :

$$f(x, y) = xy(1 - x - y)$$

10. Evaluate $\iint x^2 y^2 \, dx\, dy$ over the region $x^2 + y^2 \leq 1$.

11. Evaluate the triple integral :

$$\iiint x^{l-1} y^{m-1} z^{n-1} \, dx\, dy\, dz$$

where $\left(\frac{x}{a}\right)^p + \left(\frac{y}{b}\right)^q + \left(\frac{z}{c}\right)^r \leq 1.$

12. Prove that :

$$\int_0^{\pi/2} \frac{dx}{\sqrt{\sin x}} \times \int_0^{\pi/2} \sqrt{\sin x} = \pi$$

13. Show that :

$$\nabla^2\left(\frac{1}{r}\right)=0$$

14. Evaluate :

$$\int_C \vec{F}.dr,$$

where $\vec{F}=x^2y^3i+5yi$

and curve C is $y^2=4x$ in the xy-plane from point (0, 0) to (4, 4).

15. Show that :

$$F=(\sin y+z)i+(x\cos y-z)j+(x-y)k$$

is a conservative field.

Section—B

(Long Answer Type Questions)

Note : Attempt any *three* questions. Each question carries 20 marks. 3×20=60

1. If $y=e^{a\sin^{-1}x}$, show that :

$$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$$

2. Find out for what value of λ , the equations :

$$x + y + z = 1$$

$$x + 2y + 4z = \lambda$$

$$x + 4y + 10z = \lambda^2$$

have a solution and solve completely in each case.

3. Transform the following integral :

$$\int_0^a \int_0^{\sqrt{a^2 - x^2}} y^2 \sqrt{x^2 + y^2}\, dx\, dy$$

by changing to polar coordinates and hence deduce it.

4. Prove that :

$$\int_S (x^2 i + y^2 j + z^2 k).n dS$$

vanishes, where S is the surface of the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1.$

5. Prove that every square matrix satisfies its own characteristic equation. Using it calculate :

$$2A^5 - 3A^4 + A^2 - 4I,$$

where $A = \begin{bmatrix} -3 & 1 \\ -1 & 2 \end{bmatrix}$.

6. Find the maximum value of $u = x^m y^n z^p$ subject to conditions $x + y + z = a$.